

Glossary

Aims of teaching mathematics: in describing the importance of mathematics in the primary curriculum a number of different kinds of aims in teaching mathematics can be identified; these can be classified as utilitarian, application, intellectual development, aesthetic and epistemological.

Acute angle: an angle between 0° and 90° .

Ad hoc subtraction: an acceptable written method for doing division calculations, in which ad hoc multiples of the divisor are subtracted from the dividend until no more can be subtracted. The method is easier to understand than long division and learners can operate at their own level of confidence with number relationships.

Adhocorithm: my term for any informal, non-standard way of doing a calculation, where the method used is dependent on the particular numbers in the problem and the relationships between them.

Aesthetic aim in teaching mathematics: mathematical experiences in the primary school can provide delight, wonder, beauty and enjoyment.

Algebra: a branch of mathematics in which letters are used to represent variables in order to express generalizations.

Algebraic operating system: a system used by scientific calculators and spreadsheet software that follows the algebraic conventions of precedence of operators.

Algorithm: in number work, a standard, written procedure for doing a calculation, which, if followed correctly, step by step, will always lead to the required result; examples of algorithms are subtraction by decomposition, long multiplication and long division.

Angle: dynamically, a measure of the amount of turn (rotation) from one direction to another; statically, the difference in direction between two lines meeting at a point.

Ante meridiem and post meridiem: abbreviated to a.m. and p.m., before noon and after noon respectively.

Application aim in teaching mathematics: mathematics has many important applications in other curriculum areas.

Area: the amount of two-dimensional space enclosed by a boundary; like the size of a field enclosed by a fence.

Areas method for multiplication: a more expanded approach to multiplication of two numbers, in which the two numbers are interpreted as the sides of a rectangle and their product is the area. In this procedure, $426 \cdot 37$ is calculated as the sum of six areas: $400 \cdot 30$, $400 \cdot 7$, $20 \cdot 30$, $20 \cdot 7$, $6 \cdot 30$ and $6 \cdot 7$.

Associative law of addition: the principle that if there are three numbers to be added it makes no difference whether you start by adding the first and second, or by adding the second and third. In symbols, this law states that, for any three numbers a , b and c , $(a + b) + c = a + (b + c)$.

Associative law of multiplication: the principle that if there are three numbers to be multiplied together it makes no difference whether you start by multiplying the first and second, or by multiplying the second and third. In symbols, this law states that, for any three numbers a , b and c , $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Augmentation: the process modelled by addition in which a given quantity is increased by a certain amount and addition is used to determine the result of the increase. This structure includes 'start at ... count on by ...'.

Average: a representative value for a set of numerical data, enabling comparisons to be made between sets; three types of average are the mean, the median and the mode.

Average speed: the total distance travelled on a journey divided by the time taken; if the distance is measured in miles and the time in hours, the average speed is given in miles per hour.

Axes (plural of axis): in a two-dimensional coordinate system, two number lines drawn at right angles to represent the variables x and y . Conventionally, the horizontal axis represents the independent variable (x) and the vertical axis the dependent variable (y).

Axiom: in mathematics, a statement that is taken to be true, usually because it is self-evident, but which cannot be proved. For example, $a + b = b + a$ for all numbers a and b .

Bar chart: a graphical representation of data, where frequencies are represented by the heights of bars or columns. (See Figure 27.6b.)

Base: the number whose powers are used for the values of the various places in the place-value system of numeration; in our system the base is ten, so the places represent powers of ten, namely, units, tens, hundreds, thousands, and so on.

Block graph: an introductory way of representing discrete data, in which each member of the population is represented by an individual square (stuck on or coloured in) arranged in columns. The frequency of a particular value of the variable is simply the number of squares in that column. (See Figure 27.6a.)

- Box-and-whisker diagram:** a pictorial representation of the five-number summary for a set of data; the inter-quartile range is represented by the width of a box, with two whiskers extending to the minimum and maximum values (see Figures 28.2 and 28.3).
- Cancelling:** the process of dividing the top number and bottom number in a fraction by a common factor to produce a simpler equivalent fraction.
- Capacity:** the volume of liquid that a container can hold; usually measured in litres and millilitres; only containers have capacity.
- Cardinal aspect of number:** the idea of a number as representing a set of things. This idea of number has meaning only in terms of non-negative integers.
- Carroll diagram:** a 2 by 2 grid used for sorting the members of a set according to whether or not they possess each of two attributes. The four cells of the grid correspond to 'yes, yes', 'yes, no', 'no, yes' and 'no, no'. (See Figure 27.4.)
- Carrying (one):** in an addition calculation the process of replacing ten in one column by one in the column to the left; for example, 10 tens are replaced by 1 hundred, which is carried to the hundreds column.
- Celsius scale (°C):** a metric scale for measuring temperature, also called the centigrade scale, where water freezes at 0 degrees and boils at 100 degrees under standard conditions; named after Anders Celsius, 1701–44, Swedish astronomer, physicist and mathematician, who devised the scale.
- Centi:** a prefix (c) denoting one hundredth; for example, a centilitre (cl) is one hundredth of a litre.
- Centimetre (cm):** one hundredth of a metre; $100\text{ cm} = 1\text{ m}$; about the width of a child's little finger.
- Centre of rotational symmetry:** the point about which a shape with rotational symmetry is rotated in order to map onto itself.
- Circle:** a two-dimensional shape consisting of all the points that are a given distance from a fixed point, called the centre of the circle.
- Circumference:** the perimeter of a circle.
- Classification:** a key process in understanding mathematics, in which some numbers or some shapes (exemplars) are recognized as sharing a specified property or satisfying some criterion, which distinguishes them from other number or shapes (non-exemplars); for example, positive whole numbers may be classified as even or odd.
- Column addition and column subtraction:** ways of setting out an addition or subtraction calculation in which the ones, tens, hundreds and thousands (and so on) in the numbers in the calculation are arranged in columns.

- Commutative law of addition:** the principle that the order of two numbers in an addition calculation makes no difference to their sum. In symbols, the commutative law of addition states that, whatever the numbers a and b , $a + b = b + a$.
- Commutative law of multiplication:** the principle that the order of two numbers in a multiplication calculation makes no difference. For example, $5 \cdot 7 = 7 \cdot 5$. In symbols, the commutative law of multiplication states that, whatever the numbers a and b , $a \cdot b = b \cdot a$.
- Comparison:** the process modelled by subtraction in which two quantities are compared and subtraction is used to find the difference, or how much greater or less one is than the other.
- Compensation:** a strategy that involves replacing a number in a calculation with an easier number close to it and then compensating for this later. For example, to subtract 38 you could subtract 40 instead and compensate by adding on the additional 2 at the end.
- Complement of a set:** all the things in the universal set that are not within a given set. For example, the complement of the set of 7-year-olds in a class is the set of all those who are not 7 years old.
- Composite (rectangular) number:** a natural number that has more than 2 factors. A composite number can be illustrated as a rectangular array with more than one row; for example, 21 is a composite number (with factors 1, 3, 7 and 21) and can be arranged as 3 rows of 7. All non-prime numbers except 1 are composite.
- Cone:** a solid shape consisting of a circular base and one continuous curved surface tapering to a point (the apex) directly above the centre of the circular base.
- Congruence:** any transformation of a shape that leaves unchanged the lengths and angles; congruences are translations, rotations, reflections and combinations of these.
- Congruent shapes:** two or more shapes that can be transformed into each other by congruences.
- Conjecture:** an assertion the truth of which has not yet been established or checked by the individual making it.
- Connections model:** a model for understanding number and number operations, expressed in terms of the learner making cognitive connections between language, symbols, pictures and practical/real-life experiences.
- Conservation in measurement:** the principle that a measurement remains the same under certain transformations. For example, the length of an object is conserved when its position is altered; the volume of water is conserved when it is poured from one container to another.

Conservation of number: the principle that a number remains the same under certain transformations; for example, the number of items in a set does not change when the items are rearranged or spread out.

Constant difference method: an informal, ad hoc method for doing subtraction calculations, based on the idea that the difference between the two numbers does not change if you add the same number to both or subtract the same number from both.

Constant ratio method for division: a method for simplifying a division calculation by multiplying the dividend and the divisor by the same thing, or by dividing them by the same thing, thus keeping the ratio the same. For example, $180 \div 15$ can be simplified to $360 \div 30$, by doubling both numbers; and $360 \div 30$ can then be simplified to $36 \div 3$, by dividing both numbers by 10.

Continuous variable: a variable that can take any value on a continuum. For example, 'the height of the children in my class' is a continuous variable. When the value of this variable for a particular child has changed it will have done so continuously, passing through every real number value on the way.

Convergent thinking: the kind of thinking involved in seeking the one and only correct answer to a mathematical question.

Coordinates: starting from the origin, the distance moved in the x -direction followed by the distance moved in the y -direction to reach a particular point; recorded as (x, y) .

Correlation: a measure in statistics of the extent to which two variables are related or dependent upon each other.

Counter-example: a specific instance that shows a generalization to be false.

Creativity in mathematics: identified by overcoming fixations and rigidity in thinking; by divergent thinking, fluency, flexibility and originality in the generation of responses to mathematical situations.

Cube (shape): a solid shape with six square faces and all its edges equal in length.

Cube number: a number that can be represented as an arrangement of cubic units in the shape of a cube; a number that is obtained by multiplying a whole number by itself and by itself again. Cube numbers are 1, 8, 27, 64, 125, 216 ...

Cube root: the cube root of a given number is the number which when cubed gives that number; for example, because $5^3 = 125$, the cube root of 125 is 5. In symbols, this is written $\sqrt[3]{125} = 5$.

Cubed: 'to the power of 3'. For example, 'five cubed' is written 5^3 and is equal to $5 \cdot 5 \cdot 5$.

- Cubic centimetre (cm³):** the volume of a cube of side one centimetre; written 1 cm³ but read as 'one cubic centimetre'.
- Cubic metre (m³):** the SI unit of volume; the volume of a cube of side 1 metre; written 1m³ but read as 'one cubic metre'.
- Cubic unit:** a cube shape used as a measure of volume; for example, a cube made up of 5 layers of 5 rows of 5 cubic units has a volume of 125 cubic units.
- Cuboid:** a rectangular prism; a six-faced polyhedron in which any two opposite faces are identical rectangles. A cube is a cuboid in which all the faces are square.
- Cylinder:** a three-dimensional shape, like a baked-bean tin, consisting of two identical circular ends joined by one continuous curved surface.
- Deci:** a prefix (d) denoting one tenth; for example, a decilitre (dl) is one tenth of a litre.
- Decile:** the values that separate into ten parts a large set of data arranged in order of size. To say that a child's score in a standardized test is at the 7th decile, for example, means that the top three tenths of children obtained this score or better.
- Decimal point:** a punctuation mark (.) required when the numeration system is extended to include tenths, hundredths and so on; it is placed between the digits representing units and tenths.
- Decomposition (subtraction):** the column method of subtraction which uses the principle of exchange to overcome the difficulty caused when the digit to be subtracted in a particular column is less than the one it is being subtracted from; for example, if there are not sufficient tens in the top number to do the subtraction in that column, one of the hundreds is exchanged for 10 tens.
- Deductive reasoning:** reasoning based on logical deductions.
- Degree:** a measure of angle; 360 degrees (360°) is a complete turn.
- Denominator:** the bottom number in a fraction.
- Dependent variable:** in a relationship between two variables, the one whose values are determined by the value of the independent variable and the rule.
- Diameter:** a line from any point on a circle, passing through the centre to the point opposite; also the length of such a line; twice the radius.
- Digital root:** the result of finding the digital sum of the digital sum of a natural number repeatedly until a single digit answer is obtained; for example 8937 has a digital sum of 27 (because $8 + 9 + 3 + 7 = 27$) and therefore a digital root of 9 (because $2 + 7 = 9$).
- Digital sum:** the sum of all the digits in a given natural number; for example, the digital sum of 8937 is 27 ($8 + 9 + 3 + 7$).

Digits: the individual symbols used to build up numerals in a numeration system; in our Hindu-Arabic system the digits are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Direct proportion: the relationship between two variables where the ratio of one to the other is constant. For example, the number of cows' legs in a field and the number of cows would normally be in direct proportion.

Discrete sets: two (or more) sets that do not overlap, having no members in common; for example, the set of boys and the set of girls in a class are discrete sets.

Discrete variable: a variable that can take only specific, separate (discrete) values. For example, 'number of children in a family' is a discrete variable, because it can take only the values 0, 1, 2, 3, 4, and so on. When the value of this variable changes it goes up in jumps.

Distributive laws of division: the laws that allow you to distribute a division across an addition or across a subtraction. For example, $92 \div 4$ can be split up into $80 \div 4$ add $12 \div 4$, or $100 \div 4$ subtract $8 \div 4$. Formally, the laws state that for any numbers, a , b and c (provided c is not zero), then $(a + b) \div c = (a \div c) + (b \div c)$ and $(a - b) \div c = (a \div c) - (b \div c)$.

Distributive laws of multiplication: the laws that allow you to distribute a multiplication across an addition or across a subtraction. For example, $28 \cdot 4$ can be split up into $25 \cdot 4$ add $3 \cdot 4$, or $30 \cdot 4$ subtract $2 \cdot 4$. Formally, the laws state that for any numbers, a , b and c , then $(a + b) \cdot c = (a \cdot c) + (b \cdot c)$ and $(a - b) \cdot c = (a \cdot c) - (b \cdot c)$.

Divergent thinking: the opposite of convergent thinking; thinking characterized by flexibility, generating many different kinds of response in an open-ended task.

Edge: the intersection of two surfaces; in particular, the straight line where two faces of a polyhedron meet.

Empty number line: a number line without a scale, used to support mental and informal additions and subtractions; numbers involved in the calculation can be placed anywhere on the line provided they are in the right order relative to each other.

Epistemological aim in teaching mathematics: mathematics should be learnt because it is a distinctive and important form of knowledge and part of our cultural heritage.

Equal additions: a formal procedure for doing subtraction calculations, based on the idea of adding 10 or 100 or 1000 (and so on) to both numbers, thus keeping the difference the same; the method is no longer taught in British primary schools because decomposition is easier to understand in terms of the manipulation of base-ten materials.

Equal sharing: the process modelled by division in which a set of items or a given quantity is shared equally between a number of individuals; the key phrase in the equal-sharing structure of division is 'shared equally between'.

Equation: a statement of equivalence involving one or more variables, which may or may not be true for any particular value of the variable(s). To solve an equation is to find all the values of the variable(s) that make the equivalence true. For example, $2x + 1 = 16 - x$ is an equation with the solution $x = 5$.

Equilateral triangle: a triangle with all three sides equal in length; the three angles are also equal, and each one is therefore 60° .

Equivalence: the mathematical term for any relationship in which one mathematical entity (number, shape, set, and so on) in some sense is the same as another; in identifying an equivalence we focus on what is the same, regardless of how the entities are different.

Equivalent fractions: two or more fractions that represent the same part of a unit or the same ratio. For example, $\frac{2}{3}$, $\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$ are all equivalent fractions.

Equivalent ratios: different ways of expressing the same ratio; for example the ratio 30:50 can be written as the equivalent ratio 3:5.

Evens: where we judge an event to be as likely to happen as not to happen; probability = 0.5.

Exchange: the principle at the heart of our place-value system of numeration, in which ten in one place can be exchanged for one in the next place to the left, and vice versa; for example, 10 hundreds can be exchanged for 1 thousand, and 1 thousand can be exchanged for 10 hundreds.

Expected value: a measure used in assessing risk. A simple model for expected value of an action is the product of the probability of success and the value of the reward associated with it.

Experimental probability: an estimate of the probability of an event occurring, obtained from repeating an experiment a large number of times and finding the ratio of the number of times an event occurs to the total number of trials.

Face: one of the plane surfaces of a polyhedron.

Factor: a natural number by which a given natural number can be divided exactly, without a remainder; for example, 7 is a factor of 28.

Fahrenheit scale ($^\circ\text{F}$): a non-metric scale for measuring temperature, where water freezes at 32 degrees and boils at 212 degrees under standard conditions; named after the inventor, Gabriel Fahrenheit, 1686–1736, a German physicist.

Fibonacci sequence: a sequence of numbers in which each term is obtained by the sum of the two previous terms. Starting with 1, the sequence is 1, 1, 2, 3, 5, 8, 13, 21 ...

First quadrant: the quadrant consisting of all those points with positive coordinates.

Five-number summary: a way of summarizing a set of numerical data by giving the minimum, the lower quartile, the median, the upper quartile and the maximum.

Formula: an algebraic rule involving one or more independent variables, used to determine the value of a dependent variable; also a rule entered into a cell in a spreadsheet to determine its value.

Fraction: a way of (a) representing a part of a whole or unit, (b) representing a part of a set, (c) modelling a division problem, (d) expressing a ratio.

Frequency table: a table recording the frequencies of each value of a variable. (See Figure 27.5.)

Frequency: the number of times something occurs within a population.

'Friendly' numbers: two numbers that are related to each other in a way that makes a calculation particularly easy; for example, $457 - 257$. Often a calculation can be made easier by replacing one of the numbers with a more friendly number close to it and then compensating later.

Front-end approach: a method for doing a calculation that focuses first on the digits at the front of the number. For example, to add 543 and 476, a front-end approach would start by adding the 500 and 400.

Function: in a mapping, the relationship between the dependent variable and the independent variable. For example, if $y = 2x + 1$, then y is a function of x .

Generalization: in mathematics an assertion that something is true for all the members of a set of numbers or shapes or people. A generalization may be true (for example, 'all multiples of 12 are multiples of 3') or false (for example, 'no women can read maps').

gregation: the process modelled by addition in which two quantities are combined into a single quantity and addition is used to determine the total. The key language is 'how many altogether?'

Global generalization: when the input and output sets of a mapping are tabulated, a rule for getting the value of the dependent variable from any value of the independent variable; the left-to-right rule.

Goal difference: in football, the number of goals scored by a team subtract the number of goals scored against them; this can therefore be either a positive integer or a negative integer. This is an unconventional use of the word 'difference', which is usually just given as a positive number.

Grid method for multiplication: an alternative way of recording the steps in the areas method, without actually drawing the rectangle.

Grouped discrete data: data arising from a discrete (usually numerical) variable where the different values of the variable have been grouped into intervals, in order to reduce the number of subsets. For example, marks out of 100 in a mathematics test might be grouped into intervals 1–10, 11–20, 21–30, 31–40, and so on.

Highest common factor: for two (or more) natural numbers the highest number that is a factor of both (or all) of them; for example, the highest common factor of 24 and 36 is 12. For ‘factor’ see Glossary for Chapter 11.

Hypotenuse: the longest side of a right-angled triangle.

Hypothesis: a generalization that someone might make, which they have yet to prove to be true in every case.

Imperial units: units of measurement that were at one time statutory in the UK, most of which have now been officially replaced by metric units.

Improper fraction: a fraction in which the top number is greater than the bottom number; a fraction greater than 1; informally, a top-heavy fraction.

Independent events: two (or more) events where whether or not one occurs is completely independent of the other; for example, throw 6 on the red die, throw 6 on the blue die. The probability of both of two independent events occurring is the product of their individual probabilities.

Independent variable: in a relationship between two variables, the variable whose values may be chosen freely from the given input set, and are then put into the rule to generate the values of the dependent variable in the output set.

Inductive reasoning: in mathematics, the process of looking at a number of specific instances that are seen to have something in common and then speculating that this will always be the case.

Inequality: a statement that one number is greater than another ($>$) or less than another ($<$). For example, $80 < 87$ (80 is less than 87) and $100 > 87$ (100 is greater than 87).

Integer: a whole number, positive, negative or zero.

Intellectual development aim in teaching mathematics: mathematics provides opportunities for developing important intellectual skills in problem solving, deductive and inductive reasoning, creative thinking and communication.

Inter-quartile range: the difference between the upper and lower quartiles; a measure of spread, not affected by what happens at the extremes.

Intersection of two sets: the set of all those things that are common to the two sets. In a Venn diagram the intersection is represented by the overlap between two enclosed regions. The intersection of the set of girls and the set of 7-year-olds is the set of 7-year-old girls.

Interval scale: a measuring scale in which two measurements can be meaningfully compared only by their difference, not by their ratio; for example, if the temperature outside is -3°C and the temperature inside is $+15^{\circ}\text{C}$ then the temperature difference of 18°C is the only sensible comparison to make.

Inverse of addition: the process modelled by subtraction in which the question asked is ‘what must be added?’ in order to reach some target.

Inverse of multiplication: the process modelled by division in which the question is ‘how many groups of a given number are there in a given set?’ For example, ‘how many 4s make 20?’ corresponds to $20 \div 4$.

Inverse processes: two processes, one of which has the effect of undoing the effect of the other. For example: add 7 and subtract 7; double and halve; turn clockwise through a right angle and turn anticlockwise through a right angle.

Irrational number: a number that is not rational; for example $\sqrt{2}$ is irrational because it cannot be written exactly as one whole number divided by another.

Isosceles triangle: a triangle with two equal sides; the two angles opposite these two equal sides are also equal.

Kilo: a prefix (k) denoting a thousand; for example, a kilometre (km) is one thousand metres.

Kilogram (kg): the SI unit of mass; equal to 1000 grams.

Lies between: this phrase when used for comparing numbers or quantities can be expressed using two ‘less than’ symbols or two ‘greater than’ symbols. For example, ‘87 lies between 80 and 100’ could be written $80 < 87 < 100$ or $100 > 87 > 80$.

Line graph: mainly used for statistical data collected over time; the frequencies (or other measurements) are plotted as points and each point is joined to its neighbours by straight lines. (See Figure 27.12.) A line graph is therefore useful for showing trends over time.

Line of symmetry: the mirror line in which a shape with reflective symmetry is reflected onto itself.

Linear relationship: a relationship between two variables that produces a straight-line graph. If the two variables are directly proportional the straight line passes through the origin.

Litre: a unit used to measure liquid volume and capacity; equal to 1000 cubic centimetres. The mass of a litre of water is 1 kilogram.

Long division: a condensed and formal written method for division by two-digits numbers (and larger). The procedure is difficult to understand and involves some tricky multiplications; learners have to be able to recall accurately a complicated sequence of steps.

Long multiplication: a condensed and formal written algorithm for multiplying two numbers, based on the distributive law for multiplication. In this procedure, for example, $426 \cdot 37$ is calculated in two steps, $426 \cdot 30$ and $426 \cdot 7$.

- Lower quartile (LQ):** if the items in a set of numerical data are arranged in order of size from smallest to largest, the position of the lower quartile is $\frac{1}{4}$ of $(n + 1)$.
- Lowest common multiple:** for two (or more) natural numbers the smallest number that is a multiple of both (or all) of them.
- Mapping:** a system consisting of an input set, a rule and an output set.
- Mass:** a measurement of the quantity of matter in an object, measured, for example, in grams and kilograms; technically not the same thing as weight.
- Mathematical modelling:** the process of moving from a problem in the real world, to a mathematical model of the problem, then obtaining the mathematical solution, interpreting it back in the real world, and finally checking the result against the constraints of the original problem.
- Mean (arithmetic mean):** for a set of numerical data the result of adding up all the numbers in the set and dividing by the number in the set.
- Meaningful learning mind set:** a commitment in the learner to making sense of new material, to understanding it, by making cognitive connections with existing understanding; a preference for understanding rather than just learning by rote.
- Median:** the value of the one in the middle when all the items in a set of numerical data are arranged in order of size. If the set has an even number of items, the median comes halfway between the two in the middle. In a set of n items arranged in order, the position of the median is $\frac{1}{2}$ of $(n + 1)$.
- Metre (m):** the SI unit of length; about the distance from my nose to my fingertip when my arm is outstretched.
- Milli:** a prefix (m) denoting one thousandth; for example, a millilitre (ml) is one thousandth of a litre.
- Minus and Plus:** synonyms for 'subtract' and 'add' respectively. Strictly speaking, it is incorrect to refer to negative integers and positive integers as 'minus numbers' and 'plus numbers', as is often done by weather forecasters.
- Mixed number:** a way of writing a fraction greater than 1 as a whole number plus a proper fraction. For example, $\frac{18}{5}$ as a mixed number is $3\frac{3}{5}$ (three and three-fifths).
- Mode:** the value in a set of numerical data that occurs most often; a type of average only appropriate for large sets with a relatively small number of possible values.
- Multiple of 10:** a number that can be divided exactly by 10. So the multiples of 10 are 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, and so on. Similarly, multiples of 100 are 100, 200, 300, and so on.
- Multiples of a number:** the set of numbers obtained by multiplying a given natural number by each of the natural numbers in turn; for example, the multiples of 4 are

4, 8, 12, 16, 20, and so on; a multiple of 4 is therefore any number that can be divided exactly by 4.

Multiplicand: the number or quantity that is to be multiplied – a word not often used nowadays.

Multiplier: the number by which a multiplicand is multiplied.

Mutually exclusive events: two (or more) events such that if one occurs then the other cannot occur; for example, throw 6 on the blue die, throw 5 on the blue die. The probability that one or other of a number of mutually exclusive events will occur is the sum of their individual probabilities.

Natural numbers: the set of numbers that we use for counting, 1, 2, 3, 4, 5, and so on, going on for ever.

Near-double: when two numbers involved in an addition are nearly the same, such as $46 + 48$; or when one number involved in a subtraction is nearly double or half of the other, such as $87 - 43$. Such calculations can be done by treating them as exact doubles and then compensating.

Negative correlation: a correlation such that when one variable is low the other tends to be high, and vice versa; indicated by points tending to be clustered around a downward-sloping line in a scattergram.

Negative integer: a number less than zero. The integer -4 is correctly referred to as 'negative four'.

Net: a two-dimensional arrangement of shapes that can be cut out and folded up to make a polyhedron.

Newton: the SI unit of force (and therefore of weight); a newton is the force required to make a mass of 1 kg accelerate at the rate of one metre per second per second; named after Sir Isaac Newton, 1642–1727, English scientist and mathematician.

Number line: a straight line in which points on the line are used to represent numbers, emphasizing particularly the order of numbers and their positions in relation to each other.

Numeral: the symbol used to represent a number; for example, the number of children in a class might be represented by the numeral 30.

Numerator: the top number in a fraction.

Oblong rectangle: a rectangle that is not a square.

Obtuse angle: an angle between 90° and 180° .

Order of rotational symmetry: the number of ways in which a shape can be mapped onto itself by rotations of up to 360° . For example, a square has rotational symmetry of order 4.

- Ordinal aspect of number:** the idea of a number as representing a point on a number line. This idea of number as a label for putting things in order has meaning for negative as well as positive numbers.
- Origin:** the point where the axes in a coordinate system cross; the point with coordinates (0, 0).
- Parallel lines:** two lines drawn in the same plane, which, if continued indefinitely, would never meet.
- Parallelogram:** a quadrilateral with opposite sides parallel and equal in length.
- Partitioning (subtraction structure):** the process modelled by subtraction in which a quantity is partitioned off or taken away from a given quantity and subtraction is used to determine how many are left (or how much is left). The key idea is 'take away ... how many (much) left?'
- Partitioning into hundreds, tens and ones:** breaking a number up into hundreds, tens and ones as an aid to using it in a calculation. For example, 476 when partitioned is $400 + 70 + 6$.
- Pentagon, hexagon, heptagon, octagon, nonagon, decagon:** polygons with, respectively, five, six, seven, eight, nine and ten sides (and angles).
- Per cent (%):** in (or 'for') each hundred; for example, 87% means 87 in each hundred.
- Per:** an important word in multiplication and division situations, meaning 'for each'; used, for example, in problems about cost per unit of measurement.
- Percentage increase or percentage decrease:** an increase or decrease expressed as a percentage of the original value.
- Percentile:** the values that separate into 100 parts a large set of data arranged in order of size. To say that a child's score in a standardized test is at the 95th percentile, for example, means that the top 5% of children obtained this score or better.
- Perimeter:** the total length all the way round a boundary enclosing an area; like the length of fencing enclosing a field.
- Pi (π):** a number equal to the ratio of the circumference of any circle to its diameter; about 3.14.
- Pictogram:** a way of representing discrete data, in which each member of the population is represented by an individual picture or icon arranged in rows or columns. (See Figure 27.10.) With larger populations, each picture or icon may represent a number of individuals rather than just one.
- Pie chart:** a way of representing statistical data where the population is represented by a circle (the pie) and each subset is represented by a sector of a circle

(a slice of the circular pie), with the size of each sector indicating the frequency. (See Figure 27.11.)

Place holder: the role of zero in the place-value system of numeration; for example, in the numeral 507 the 0 holds the tens place to indicate that there are no tens here. Without the use of zero as a place holder there would just be a gap between the 5 and the 7.

Place value: the principle underpinning the Hindu-Arabic system of numeration in which the position of a digit in a numeral determines its value; for example, '6' can represent six, sixty, six hundred, six tenths, six hundredths, and so on, depending on where it is written in the numeral.

Plane of symmetry: a plane that cuts a solid shape into two halves that are mirror images of each other.

Plane surface: a completely flat surface; any two points on the surface can be joined by a straight line drawn on the surface.

Polygon: a two-dimensional closed shape, consisting of straight sides.

Polyhedron (plural, polyhedra): a three-dimensional shape with only straight edges and plane surfaces.

Population: the term used in statistics for the complete set of all the people or other things for which some statistical data is being collected; synonymous with 'universal set' in set theory.

Positive correlation: a correlation such that when one variable is high the other tends to be high, and when one is low the other tends to be low; indicated by points tending to be clustered around an upward-sloping line in a scattergram.

Positive integer: an integer greater than zero. The integer +4 is correctly referred to as 'positive four'. Usually the + sign is understood and the integer is just written as 4 and referred to as 'four'.

Power: a way of referring to a number repeatedly multiplied by itself; for example, $10 \cdot 10 \cdot 10 \cdot 10$ is referred to as '10 to the power 4', abbreviated to 10^4 .

Precedence of operators: a convention that, unless otherwise indicated by brackets, the operations of multiplication and division should have precedence over addition and subtraction. This convention is always used in algebraic expressions.

Prime factorization: writing a given natural number as the product of prime numbers; for example, the prime factorization of 63 is $3 \cdot 3 \cdot 7$. Each composite number has a unique prime factorization.

Prime number: a natural number that has precisely two factors (namely, 1 and itself). The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

- Prism:** a polyhedron consisting of two opposite identical faces with their vertices joined by parallel lines.
- Pro rata increase:** an increase applied, for example, to salaries or prices, in which each amount is increased by the same scale factor.
- Probability scale:** a scale for measuring probability, ranging from 0 (impossible) to 1 (certain).
- Probability:** a mathematical measure of the strength of our belief that some event will occur, based on whatever evidence we can assemble; a measure of how likely an event is to happen.
- Problem:** in mathematics, a situation consisting of some givens and a goal, with a cognitive gap between them; this constitutes a problem for an individual if the way to fill the gap between the givens and the goal is not immediately obvious.
- Product:** the result of a multiplication; for example, the product of 37 and 27 is 999.
- Proof by exhaustion:** a method for proving a generalization by checking every single case to which it applies.
- Proof:** a complete and convincing argument to support the truth of an assertion in mathematics, which proceeds logically from the assumptions to the conclusion.
- Proper fraction:** a fraction in which the top number is smaller than the bottom number; a fraction less than 1.
- Proportion:** a comparative part of a quantity or set. A proportion (such as 4 out of 10) can be expressed as a fraction ($\frac{2}{5}$), as a percentage (40%) or as a decimal (0.4).
- Protractor:** a device for measuring angles.
- Pyramid:** a polyhedron consisting of a polygon as a base, with straight lines drawn from each of the vertices of the base to meet at one point, called the apex.
- Pythagorean triple:** three natural numbers that could be the lengths of the three sides of a right-angled triangle. For example, 5, 12 and 13 form a Pythagorean triple because $5^2 + 12^2 = 13^2$.
- Quadrant:** One of the four regions into which the plane is divided by the two axes in a coordinate system.
- Quadrilateral:** a plane shape with four straight sides and four interior angles. The four angles of any quadrilateral add up to 360° .
- Quartiles:** the three items in a set of numerical data arranged in order of size, from smallest to largest, that come one quarter of the way along (the lower quartile), in the middle (the median), and three quarters of the way along (the upper quartile).

Quotient, divisor and dividend: technical names for the three numbers involved in a division calculation; for example, in $999 \div 37 = 27$, the 999 is the dividend, 37 is the divisor and 27 is the quotient.

Radius: a line from the centre of a circle to any point on the circle; also the length of such a line; half the diameter.

Range: in a set of numerical data, the difference between the largest and smallest value; a simple measure of spread that can be used to compare two sets of data.

Ratio scale: a measuring scale in which two measurements can be meaningfully compared by ratio; for example, if the mass of one object is 30 kg and the mass of another is 10 kg then it makes sense to say that one is 3 times heavier than the other.

Ratio: the inverse of the scaling structure of multiplication, where division is used to compare two quantities; for example, the ratio of £36 to £12 is 3; this is represented by the division $36 \div 12 = 3$.

Rational number: a number that can be expressed as the ratio of two integers (whole numbers). All whole numbers and fractions are rational numbers, as are all numbers that can be written as exact decimals.

Real number: any number that can be represented by a length or by a point on a continuous number line. The set of real numbers consists of all rational and all irrational numbers.

Rectangle: a parallelogram in which all four of the angles are right angles. A square is a rectangle with all sides equal in length.

Rectangular array: a set of objects or shapes arranged in rows and columns, in the shape of a rectangle; for example, 7 rows of 5 counters, or a 7 by 5 grid of squares. Rectangular arrays are important images to be associated with multiplication.

Recurring decimal: a decimal, which might be the result of a division calculation, where one or more digits after the decimal point repeat over and over again, for ever. For example, $48 \div 11$ is equal to 'four point three six recurring' (4.36363636 ... with the '36' being repeated over and over again, for ever).

Reduction: the process modelled by subtraction in which a given quantity is reduced by some amount and subtraction is used to determine the result of the reduction. This structure includes 'start at ... count back by ...'.

Reference item: a measurement that is memorized and used as a reference point for estimating other measurements; for example, the capacity of a wine bottle is 750 ml.

Reflection: a transformation in which a shape is reflected in a mirror line and changed into its mirror image.

Reflective symmetry: the property possessed by a shape that is its own mirror image; also called line symmetry.

Reflex angle: an angle between 180° and 360° .

Regular polygon: a polygon in which all the sides are equal in length and all the angles are equal in size.

Regular polyhedron: a polyhedron in which all the faces are identical shapes, the same number of edges meet at each vertex in an identical configuration, and all the edges are equal in length.

Relative frequency: an estimate of the probability of an event occurring in the members of a population, obtained from the ratio of the number of times an event is recorded in a sample to the total number in the sample.

Remainder: in a division situation that does not work out exactly, the surplus number after an equal sharing or grouping has been completed. For example, $45 \div 7 = 6$, remainder 3. This could mean 45 shared equally between 7 gives 6 each, with 3 not shared out; or it could mean that 6 subsets of 7 can be made from a set of 42, with 3 not in a subset.

Repeated addition (division): one of the ways of experiencing the inverse-of-multiplication structure of division by repeatedly adding a quantity to reach a given target; for example, 'how many payments of £6 are required to make a total of £24?' is connected with $24 \div 6$.

Repeated aggregation: the process modelled by multiplication related to the idea of 'so many sets of so many'; also called repeated addition.

Repeated subtraction (division): one of the ways of experiencing the inverse-of-multiplication structure of division by repeatedly subtracting a quantity from a given amount; for example, 'how many times can £6 be taken away from £24 until there is nothing left?' is connected with $24 \div 6$.

Rhombus: a parallelogram in which all four sides are equal in length; a diamond. A square is a rhombus with all four angles equal.

Right angle: an upright angle, a quarter-turn, 90° .

Rotation: a transformation in which a shape is rotated through an angle about a centre of rotation; every line in the shape turns through the same angle.

Rotational symmetry: the property possessed by a shape that can be mapped exactly onto itself by a rotation (other than through a multiple of 360°).

Rote-learning mind set: a tendency in a learner to learn new material as isolated pieces of knowledge or skills, without making cognitive connections with existing

networks of connections; a preference for relying on memorization and recall, rather than seeking to understand.

Rounding: in Chapter 5, transforming an answer that is not an exact whole number into a whole number, either the whole number above (rounding up) or the one below (rounding down). (See also Chapter 13.)

Rounding: (Chapter 13) the process of approximating an answer to a calculation to an appropriate degree of accuracy; this can be done by rounding up or rounding down or rounding to the nearest something. For example, £25.37 rounded up to the next ten pence is £25.40 and rounded down to the next ten pence is £ 25.30; rounded to the nearest ten pence it is £25.40, because it is nearer to this than to £25.30.

Sample: in a statistical survey a representative selection of a large population for which data is collected; in general, the larger the sample, the more reliable are the results as a representation of the whole population.

Scalene triangle: a triangle with all the three sides different in length.

Scaling down: scaling by a factor less than 1 (but greater than zero).

Scaling of quantity: the process modelled by multiplication in which a given quantity is increased by a scale factor; doubling and trebling are examples of scaling by factors of 2 and 3, respectively. Scaling by a factor less than 1 (for example, halving) reduces the size of the quantity.

Scaling of shape: a transformation in which all the lengths in a shape are multiplied by the same factor, called the scale factor; the angles remain unchanged.

Scaling up: scaling by a factor greater than 1.

Scatter diagram (scatter plot, scattergram, scattergraph): a graphical representation of data for two variables for a given set, with horizontal and vertical axes representing the two variables, and the values of the two variables for each individual in the set plotted as points.

Scientific notation (standard form): representing a number, especially a very large or a very small one, as a number between 1 and 10 multiplied by a power of 10. For example, 7 654 000 would be written as $7.654 \cdot 10^6$ and 0.000 765 4 would be written as $7.654 \cdot 10^{-4}$.

Separator: the function of the decimal point in the contexts of money and other units of measurements, where it serves to separate, for example, pounds from pence, or metres from centimetres..

Sequential generalization: when the input and output sets of a mapping are tabulated, a rule for getting the next value of the dependent variable from the previous one(s); the up-and-down rule.

- Short division:** a compact standard algorithm for a division calculation involving a single-digit divisor. The divisor is divided into each digit in turn, working from left to right, with any remainders being transferred to the next column.
- SI units:** an agreed international system of units for measurement, based on one standard unit for each aspect of measurement.
- Significant figures:** the digits in a number as you read from left to right; for example, in 25.37 the first significant figure is the 2, then the 5, then the 3 and then the 7. This number rounded to two significant figures is 25 and rounded to three significant figures is 25.4.
- Similar shapes:** two shapes either one of which is a scaling of the other. In similar shapes, corresponding lines are in the same ratio and corresponding angles are equal.
- Special case:** a specific instance that does not fit an otherwise true generalization and may have to be removed from the set to which the generalization is applied.
- Sphere:** a completely round ball; a solid shape with one continuous surface, in which every point on the surface is the same distance from a point inside the shape called the centre.
- Spreadsheet:** on a computer, a rectangular array of cells, labelled by rows and columns (for example, cell B3 is in column B and row 3), into which data can be entered; the data can be words, numbers or formulas.
- Square (shape):** a rectangle (see Chapter 25) with all four sides equal in length.
- Square centimetre (cm²):** the area of a square of side one centimetre; written 1 cm² but read as 'one square centimetre'. There are ten thousand square centimetres in a square metre.
- Square metre (m²):** the SI unit of area; the area of a square of side one metre; written 1 m² but read as 'one square metre'.
- Square millimetre (mm²):** the area of a square of side one millimetre; written 1 mm² but read as 'one square millimetre'. There are a million square millimetres in a square metre.
- Square number:** a number that can be represented as a square array; a number that is obtained by multiplying a whole number by itself. Square numbers are 1, 4, 9, 16, 25, 36, 49, 64 ...
- Square root:** the (positive) square root of a given number is the positive number which when squared gives that number; for example, because $5^2 = 25$, the (positive) square root of 25 is 5. In symbols, this is written $\sqrt{25} = 5$.

Square unit: a square shape used as a measure of area; for example, a square made up of 5 rows of 5 square units has an area of 25 square units.

Squared: 'to the power of 2'. For example, 'five squared' is written 5^2 and is equal to $5 \cdot 5$.

Stepping stone: usually a multiple of 10 or 100 used to break down an addition or subtraction into easier steps. For example, to find what has to be added to 37 to get to 75, the numbers 40 and 70 might be used as stepping stones.

Straight angle: a half-turn, 180° .

Subjective probability: an estimate of the probability of some event occurring based on subjective judgements of the available evidence.

Subset: a set of members within a given set that have some defined attribute, or that take a particular value of a variable.

Sum: the result of doing an addition; for example, 25 is the sum of 17 and 8. The word 'sum' should not be used as a synonym for 'calculation'.

Suppression of zero: the misleading practice of starting the vertical axis in a frequency graph at a number other than zero; this gives a false impression of the relative frequencies of various values of the variable.

Surface area: the sum of the areas of all the surfaces of a solid object.

Tallying: a simple way of counting, making a mark for each item counted, with every fifth mark used to make a group of five. (See Figure 27.5.)

Tessellation: a pattern made by repeatedly fitting together without gaps a collection of identical tiles; it must be possible to continue the pattern in all directions as far as you wish.

Tetrahedron, hexahedron, octahedron, dodecahedron, icosahedron: polyhedra with, respectively, four, six, eight, twelve and twenty faces. The regular forms of these five shapes are the only possible regular polyhedra. A cube is a regular hexahedron.

Theorem of Pythagoras: in a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

Theoretical probability: an estimate of probability based on theoretical arguments of symmetry and equally likely outcomes; if there are n equally likely outcomes from an experiment then the probability of each one occurring is $1/n$.

Transformation: the mathematical term for any process which changes a mathematical entity (number, shape, set, and so on) into another; in identifying a transformation

we focus on what is different, what has changed, even though some things may still be the same.

Transitive property: a property that any given mathematical relationship may or may not possess; the property is that if A is related to B and B is related to C then it always follows that A is related to C . Each of the relationships ‘is a factor of’ and ‘is a multiple of’ is transitive.

Translation: a transformation in which a shape is slid from one position to another, without turning.

Trapezium: a quadrilateral with two sides parallel.

Trial and improvement: a procedure for finding the solution to a mathematical problem by means of successive approximations (trials) which gradually close in on the required solution.

Triangle: a plane shape with three straight sides and three interior angles. The three angles of any triangle add up to 180° .

Triangle numbers: numbers that can be arranged as triangles of dots in the way shown in Figure 15.5. The set of triangle numbers is 1, 3, 6, 10, 15, 21, 28, and so on. The eighth triangle number, for example, is the sum of the natural numbers from 1 to 8.

Truncation: this is what a calculator does when it has to cut short an answer to a calculation by throwing away some of the digits after the decimal point, because it does not have room to display them all. For example, a calculator with space for only 8 digits in the display might truncate the result 987.654321 to 987.65432.

Two-way table: a systematic way of identifying in a rectangular array all the possible combinations of the values of two variables; used in probability to identify all the possible combinations of two independent events. (See Figure 29.1.)

Union of sets: the set formed when two (or more) sets are combined to form a single set. The union of two discrete sets is an example of the aggregation structure of addition.

Universal set: the term used in set theory for the complete set of all things under consideration; synonymous with ‘population’ in statistics. In a Venn diagram the universal set is usually represented by a rectangular box.

Upper quartile (UQ): if the items in a set of numerical data are arranged in order of size from smallest to largest, the position of the upper quartile is $\frac{3}{4}$ of $(n + 1)$.

Using and applying mathematics: using the skills, knowledge, concepts and principles learnt in mathematics to solve problems, across a continuum from genuine problems in a real-life context to purely mathematical challenges;

engaging in investigations and enquiries that develop key processes in mathematical reasoning; and communicating insights, reasoning, results and conclusions with mathematical language, diagrams and symbols.

Utilitarian aim in teaching mathematics: mathematics is useful in everyday life and necessary in most forms of employment.

Variable (in statistics): an attribute that can vary from one member of a population to another, the different values of which can be used to sort the population into subsets; variables may be non-numerical (such as choice of favourite fruit) or numerical (such as the mark achieved in a mathematics test).

Variable: a quantity the value or size of which can vary; for example, the number of children in a school is a variable, whereas the number of letters in the word 'school' is not.

Venn diagram: a way of representing the relationships between various sets and subsets using enclosed regions (such as circles); children can use these for sorting experiences by placing the members of various sets or subsets within the appropriate regions. (See Figures 27.1–27.3.)

Vertex (plural vertices): in a plane geometric shape with straight sides, a point where two sides meet; similarly, for a three-dimensional shape, a point where three or more edges meet.

Volume: the amount of three-dimensional space taken up by an object; measured in cubic units, such as cubic centimetres or cubic metres.

Vulgar fraction: an archaic term for a 'common' fraction; in other words a fraction expressed as a numerator over a denominator (for example, $\frac{3}{8}$) rather than as a decimal (that is, 0.375).

Weight: the force of gravity acting upon an object and therefore properly measured in newtons; colloquially used incorrectly as a synonym for mass.

Zero hours: midnight on the 24-hour clock.